Honors Pre Calculus Summer Math Packet

Name _____ Date _____

Solve these problems on this worksheet. Show work. Your completed assignment is due on the first day of school. An in-class assessment will be given on the first Friday.

USEFUL WEB SITES:
www.purplemath.com/modules/index.htm

For problems 1-12, solve for x. Simplify all answers. Check where required.

1. $x^2 - 5x - 24 = 0$

$(x-8)(x+3) = 0$

$x - 8 = 0$ or $x + 3 = 0$

$x = \{8, -3\}$

2. $9x^2 = 18x + x^3$

$0 = x^3 - 9x^2 + 18x$

$0 = x(x-6)(x-3)$

$x = \{0, 6, 3\}$

3. $5x^2 - 1 = 6x$

$5x^2 - 6x - 1 = 0$

$x = \dfrac{6 \pm \sqrt{36 - 4(5)(-1)}}{2(5)}$

$x = \dfrac{6 \pm \sqrt{86}}{10} = \dfrac{3 \pm \sqrt{14}}{5}$

4. $3x^2 + 2x = -6$

NO REAL SOLN.

$3x^2 + 2x + 6 = 0$

$x = \dfrac{-2 \pm \sqrt{4 - 4(3)(6)}}{2(3)}$

$4 - 4(3)(6) < 0$

$x = \dfrac{-2 \pm \sqrt{-68}}{6} = \dfrac{-1 \pm i\sqrt{17}}{3}$

5. $\left[(x+5)^{\frac{3}{2}}\right]^{\frac{2}{3}} = [125]^{\frac{2}{3}}$

$x + 5 = 25$

$x = 20$

6. $\left[(2x-1)^{\frac{4}{3}}\right]^{\frac{3}{4}} = (81)^{\frac{3}{4}}$

$\left[(2x-1)^{\frac{4}{3}}\right]^{\frac{3}{4}} = 81^{\frac{3}{4}}$

$2x - 1 = 27$

$x = 14$

If $2x - 1 < 0$

$-(2x-1) = 27$

$-2x + 1 = 27$

$-2x = 26$

$x = -13$

7. $\sqrt{16 + 3x} = 2$

$16 + 3x = 4$

$3x = -12$

$x = -4$

8. $\sqrt{3x^2 - 27x + 15} + 5 = 2x$

$\sqrt{3x^2 - 27x + 15} = 2x - 5$

$3x^2 - 27x + 15 = 4x^2 - 20x + 25$

$0 = x^2 + 7x + 10$

$0 = (x + 5)(x + 2)$

$x = \{-5, -2\}$

9. $|4x - 3| - 6 = 2$

$|4x - 3| = 8$

$4x - 3 = 8$ OR $4x - 3 = -8$

$x = \frac{11}{4}$ $\qquad x = -\frac{5}{4}$

$x = \{\frac{11}{4}, \frac{-5}{4}\}$

10. $|7x + 4| + 10 = 6$

$|7x + 4| = -4$

$\emptyset$

11. $x^2 + 52 = 0$

NO REAL SOLUTION

$x = \pm 2i\sqrt{13}$

12. $3x^2 - 6x + 10 = 0$

$x = \frac{6 \pm \sqrt{36 - 4(3)(10)}}{6} = \frac{6 \pm \sqrt{-84}}{6}$

$x = \frac{3 \pm i\sqrt{21}}{3}$

13. Find all solutions to the equation $x^4 - 7x^3 + 18x^2 - 2x - 28 = 0$. Analytically verify that each of your answers is a solution (do not solely find zeros on your graphing calculator).

$x = \{-1, 2, 3 \pm i\sqrt{5}\}$

$x^2 - 6x + 14 = 0$

$x = \frac{6 \pm \sqrt{36 - 4(1)(14)}}{2} = \frac{6 \pm \sqrt{-20}}{2}$

$-1 \underline{)} \begin{array}{ccccc} 1 & -7 & 18 & -2 & -28 \\ & -1 & 8 & -26 & 28 \end{array}$

$2 \underline{)} \begin{array}{ccccc} 1 & -8 & 26 & -28 & \underline{0} \\ & 2 & -12 & 28 & \\ \hline 1 & -6 & 14 & \underline{0} \end{array}$

14. Use your graphing calculator to estimate all real zeros of this function. Round to **thousandths.**

$f(x) = 3x^4 + 4x^3 - 11x^2 - 5x + 2$
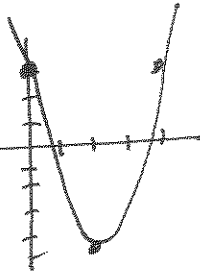
$x = \{-2.493, -.654, .264, 1.549\}$

Sketch the graphs of the equations shown in problems 15-20 without using your calculator.
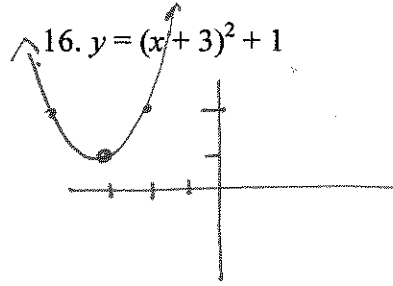
15. $y = 2x^2 - 8x + 3$

$x - \dfrac{8}{4} = 2$

$V(2, -5)$

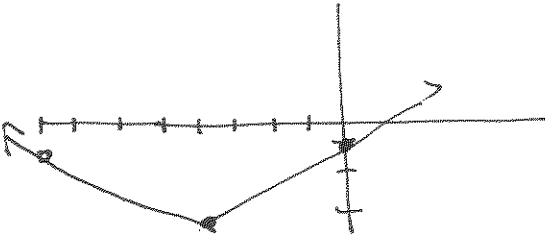| x	y
0	3
4	3

16. $y = (x + 3)^2 + 1$

Shift Left 3
Up 1

17. $y = \left|\dfrac{1}{2}x + 2\right| - 3$
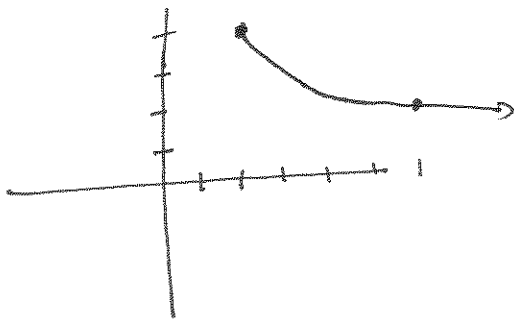
$y = \left|\dfrac{1}{2}(x + 4)\right| - 3$

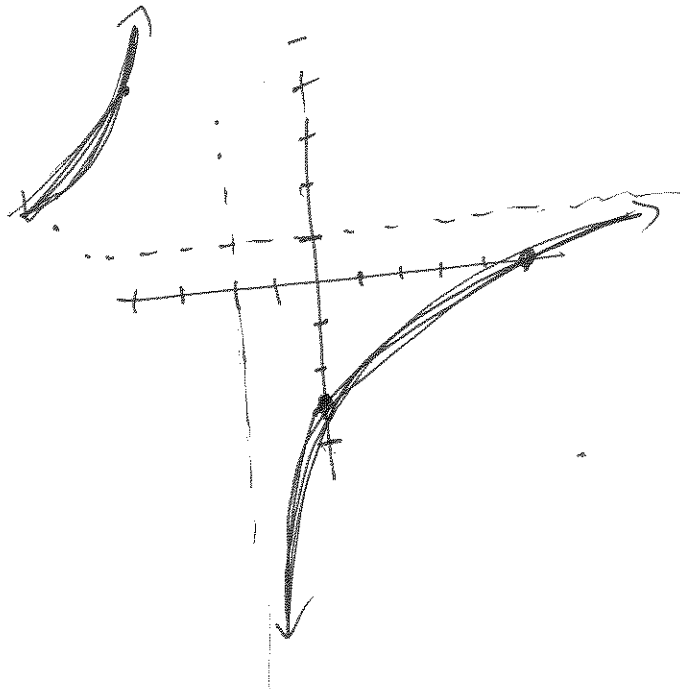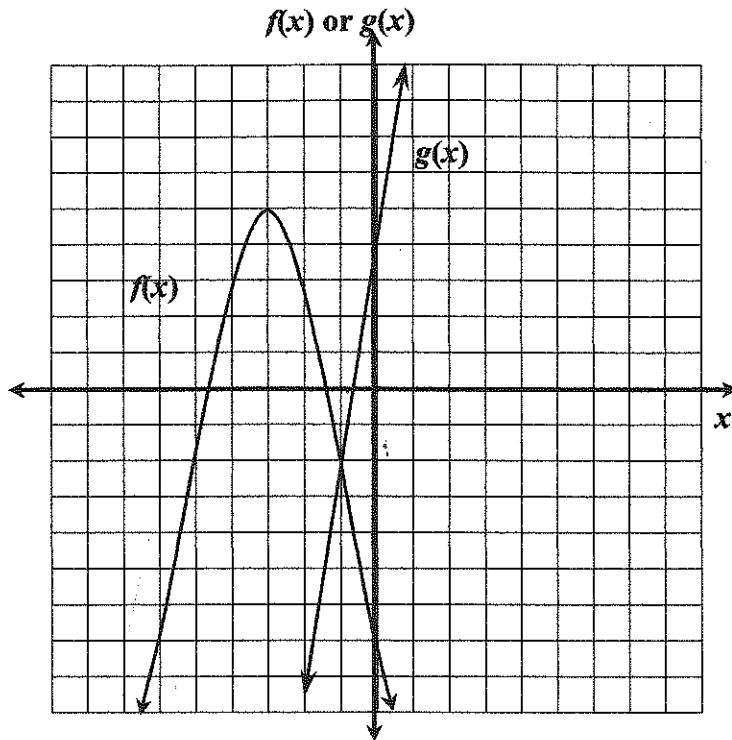| x	y
0	-1
-8	-1

18. $y = -(x + 3)(x - 1)(x - 4)^2$

40

19. $y = -\sqrt{x - 2} + 4$

20. $y = \dfrac{x - 5}{x + 2}$

Use these definitions of $f(x)$, $g(x)$, $h(x)$ and $k(x)$ to answer problems 21-24.

$f(x)$ or $g(x)$

$h(x) = 2x^2 - 4$

$k(x) = 3x + 5$

21. Evaluate each of these functions

a) $f(-2)$

2.5

b) $h(-6)$

$2(-6)^2 - 4$

68

c) $g(0)$

3.5

d) $h(k(-3))$

$h(-4)$

28

e) $h(h(2))$

28

f) $f^{-1}(5)$

-3

g) $g^{-1}(-2)$

-1

h) $h^{-1}(10)$

$2x^2 - 4 = 10$

$2x^2 - 14 = 0$

$x^2 = 7$

$x = \pm\sqrt{7}$

22. What value of a makes $f(a) = g(a)$?

$a = -1$

23. Write expressions for the following functions. Simplify your answers

a) $k(h(x))$

$3(2x^2-4)+5$
$6x^2-12+5$
$\boxed{6x^2-7}$

b) $(h \circ k)(x)$

$2(3x+5)^2-4$
$2(9x^2+30x+25)-4$
$18x^2+60x+50-4$
$\boxed{18x^2+60x+46}$

c) $k^{-1}(x)$

$\boxed{\dfrac{x-5}{3}}$

d) $2h(x)+k(x)$

$2(2x^2-4)+3x+5$
$4x^2-8+3x+5$
$\boxed{4x^2+3x-3}$

24. Give the domain and range of $f(x)$, $g(x)$, $h(x)$ and $k(x)$.

$\underline{f(x)}$

D All Reals

R $y \le 5$

$\underline{g(x)}$

All Reals

All Reals

$\underline{h(x)}$

All Reals
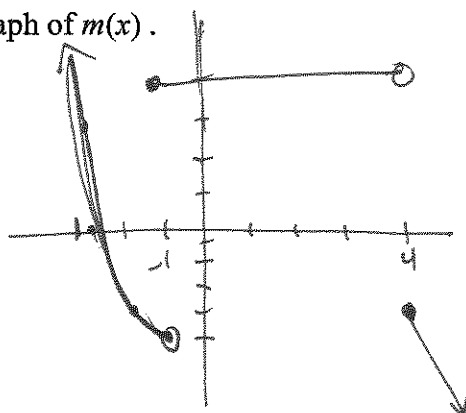
$y \ge -4$

$\underline{k(x)}$

All Reals

All Reals

Use this definition of $m(x)$ to answer questions 25-26.

$$m(x) = \begin{cases} 2x^2 + 4x - 3 & x < -1 \\ 4 & -1 \le x < 4 \\ -3x + 9 & x \ge 4 \end{cases}$$

25. Evaluate each of these functions.

a) $m(6)$

$-3(6)+9$
$\boxed{-9}$

b) $m(-2)$

$2(-2)^2+4(-2)-3$
$\boxed{-3}$

c) $m(-1)$

$\boxed{4}$

d) $m(0)$

$\boxed{4}$

26. Sketch the graph of $m(x)$.

Solve the inequalities given in problems 27-32. Give <u>exact</u> answers (not decimal approximations).

27. $10 > 6 - 2x$

$2x - 6 > -10$

$2x > -4$

$\boxed{x > -2}$

28. $|2x - 3| \leq 11$

$2x - 3 \leq 11$ AND $2x - 3 \geq -11$

$x \leq 7$ $\qquad\qquad$ $x \geq -4$

$\boxed{-4 \leq x \leq 7}$

29. $\left|\dfrac{1}{3}x - 2\right| > 3$

$\dfrac{1}{3}x - 2 > 3$ OR $\dfrac{1}{3}x - 2 < -3$

$\boxed{x > 15 \quad \text{OR} \quad x < -3}$

30. $x^2 - 4x + 3 \geq 0$

$x^2 - 4x + 3 = 0$

$(x - 3)(x - 1) = 0$

$x = 3, 1$

+++ O --- O +++
$\qquad$ 1 $\qquad$ 3

$\boxed{x \leq 1 \text{ OR } x \geq 3}$

31. $-x^2 + 3x + 2 \geq \dfrac{1}{2}x - 1$
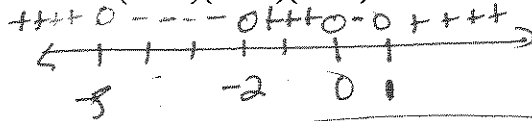
$-2x^2 + 6x + 4 \geq x - 2$

$-2x^2 + 5x + 6 \geq 0$

$2x^2 - 5x - 6 \leq 0$

$x = \dfrac{5 \pm \sqrt{73}}{4}$ $\qquad$ $\dfrac{5 - \sqrt{73}}{4} \leq x \leq \dfrac{5 + \sqrt{73}}{4}$

32. $x(x + 5)(x + 2)(x - 1) < 0$

+++ O ----- O +++ O - O +++
$\quad$ 5 $\qquad$ -2 $\quad$ 0 $\quad$

$\boxed{-5 < x < -2 \text{ OR } 0 < x < 1}$

For problems 33-42, perform indicated operations and give answers in simplest form (no calculator).

33. $\dfrac{2x\left(4x^3y^{-2}\right)^{-2}}{10y^{-3}\left(x^2y\right)^3}$

$\dfrac{2x \cdot 4^{-2} x^{-6} y^4}{10 y^{-3} x^6 y^3}$

$\dfrac{2 \cdot y^4}{10 \cdot 16 \cdot x^{11}} = \boxed{\dfrac{y^4}{80 x^{11}}}$

34. $\dfrac{2x^{\frac{3}{2}} - x^{-\frac{1}{2}}}{x^{-\frac{1}{2}}} = \dfrac{x^{-\frac{1}{2}}\left[2x^2 - 1\right]}{x^{-\frac{1}{2}}}$

$= \boxed{2x^2 - 1}$

$i^2 = -1$

$i^7 = i^4 \cdot i^3$

35. $(3-i)(6+i)$

$18 - 3i - i^2$

$\boxed{19 - 3i}$

36. $i(5i - i^6)$

$5i^2 - i^7$

$\boxed{-5 + i}$

37. $\dfrac{6-\sqrt{5}}{2+\sqrt{20}}$

$\dfrac{(6-\sqrt{5})(2-2\sqrt{5})}{(2+2\sqrt{5})\,2-2\sqrt{5}} = \dfrac{12 - 14\sqrt{5} + 10}{4 - 20}$

$\dfrac{22 - 14\sqrt{5}}{-16} = \dfrac{11 - 7\sqrt{5}}{-8} = \boxed{\dfrac{-11 + 7\sqrt{5}}{8}}$

38. $\dfrac{\left(\dfrac{8}{4-i}\right)(4+i)}{(4-i)(4+i)} \geq \dfrac{32 + 8i}{17} = \dfrac{32}{17} + \dfrac{8}{17}i$

39. $\log_3 81 = 4$

40. $\log_5 \dfrac{1}{25} = -2$

41. $\log_6 1 = 0$

42. $\ln e^4 = 4$

43. Evaluate these expressions with a calculator. Round to hundredths.

a) $\sqrt[7]{300}$

2.26

b) $10^{\frac{5}{2}}$

316.23

c) $e^{2.5}$

12.18

d) $\log 30$

1.48

e) $\log_6 50$

2.18

f) $\ln 15$

2.71

Expand these log expressions

44. $\log_4 16x^2y$

$\log_4 16 + \log_4 x^2 + \log_4 y$

$2 + 2\log_4 x + \log_4 y$

45. $\ln\left(\frac{3x}{z^2}\right)^3$

$3[\ln 3x - \ln z^2]$

$3(\ln 3 + \ln x - 2\ln z)$

Condense these log expressions

46. $\frac{1}{2}(\log x - 3\log y)$

$\frac{1}{2}\log \frac{x}{y^3}$

$\log\left(\frac{x}{y^3}\right)^{\frac{1}{2}}$

47. $4\ln x + \ln t$

$\ln x^4 t$

Solve for *x*. Round to hundredths, where necessary.

48. $4(3^x) = 21$

$$3^x = \frac{21}{4}$$

$$x \ln 3 = \ln\left(\frac{21}{4}\right)$$

$$x = \frac{\ln\left(\frac{21}{4}\right)}{\ln 3} \approx 1.51$$

49. $60\left(\frac{1}{2}\right)^{\frac{x}{83}} = 31$

$$\left(\frac{1}{2}\right)^{\frac{x}{83}} = \frac{31}{60}$$

$$\frac{x}{83} \ln\left(\frac{1}{2}\right) = \ln\left(\frac{31}{60}\right)$$

$$x = 83\left[\frac{\ln\frac{31}{60}}{\ln\frac{1}{2}}\right] \approx 79.07$$

50. $\log_x 16 = 5$

$$\left(x^5\right)^{\frac{1}{5}} = (16)^{\frac{1}{5}}$$

$$x \approx 1.74$$

51. $\log_7 3x = 4$

$$7^4 = 3x$$

$$x \approx 800.33$$